The role of working memory in mathematics learning and numeracy

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Abstract: The role of working memory in mathematics learning and numeracy. The presentation will examine the role of working memory in typical mathematics tasks, procedures for diagnosing working memory influences on mathematics learning difficulties and intervention strategies for enhancing working memory processes during mathematics learning.

Successful mathematics learning makes unique demands on working memory processes. This paper examines some of these demands and the influence of working memory on the mathematics learning. Indicators of mathematics learning difficulties attributable to working memory processes will be identified. Procedures for intervention will include cognitive strategy teaching to enhance encoding and retention and teaching for automatization of mathematics learning.

Mathematics education involves students learning to complete tasks such as 92-48 =; 1/2 + 1/3 =; or factorize 2x+8. Successful learning of these types of ideas involves the students linking ideas in particular ways, indicated in the teaching to which they have been exposed.

Some students have difficulty learning to do this. In other words, they have difficulty making use of the teaching information that allows their peers to learn the skills and understanding associated with these tasks. To what extent is this difficulty due to the need to recall what they know, retain and link ideas during learning and to encode their new understanding in long term memory ?

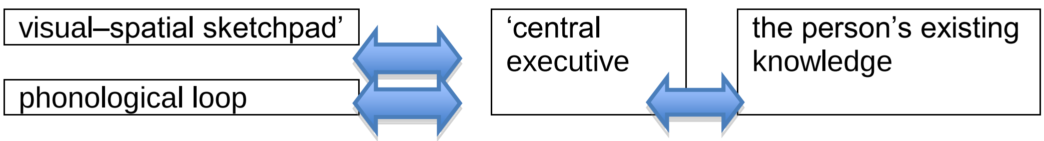
Working memory and mathematics learning difficulties

Working memory is the' mental thinking space' in which students manipulate or act on aspects of their knowledge during mathematics knowledge or while completing mathematics tasks and problems. It is the activity students engage when they interpreting teaching information using knowledge they retrieve from long term memory, when they retain and link partial mathematics ideas to synthesize new mathematics knowledge and when they direct their learning and thinking activity to compute or solve mathematical problems.

To unpack the role of working memory on mathematics learning, we will begin with Baddeley's multi-component model of working memory (Baddeley and Logie, 1999). This model identifies various processes;

1. The temporary retention of relevant mathematics information, either in a 'visual–spatial sketchpad' (for visual spatial information) or a 'phonological loop' (for retaining verbal information).
2. The manipulation of relevant ideas to form new knowledge using the appropriate cognitive activity in a 'central executive'
3. The retrieval of aspects of the person's existing knowledge, used as a foundation for interpreting the teaching information and for building new knowledge.

These components are shown in the following diagram:



It seems intuitively reasonable to expect that successful mathematics learning would require students to make efficient use of their working memory. It is perhaps not surprising for example, that the phonological loop is implicated in more tasks that involve strategy use based on counting down strategies for subtraction problems (Imbo and Vandierendonck, 2007) than in tasks that require single-digit multiplication recall from long-term memory (De Rammelaere et al., 2001). The central executive, on the other hand, usually has a greater role to play in the 'carry operation' in addition and multiplication than the phonological loop (Imbo et al., 2007).

Following a comprehensive review of the research relating to mathematics and working memory, Raghubar, Barnes and Hecht (2010) agree, but with some caveats. They note the complexity of this relationship and the likelihood that for any individual it will depend on a wide range of factors that influence how the individual interacts with the mathematics information (either the teaching information or the information specifying a problem or task). These include personal factors such as their age and skill level, mathematics content factors and characteristics of the learning – teaching context such the level of mastery being targeted (beginning, generalizing or automatizing), language of instruction and the formats in which the mathematics information presented. They note the need for 'a sufficiently comprehensive model of mathematical processing, particularly in relation to skill acquisition, that can handle current findings on working memory as well as provide the basis from which to guide new discoveries and inform practice. (page 119) '.

Children with math difficulties differ from their peers without difficulties in each aspect of their working memory processes; in verbal working memory, in static and/or dynamic visual–spatial memory processing, in numerical working memory and in backward digit span tasks. Given a lack of consistency across studies about how to measure the components of verbal and visual–spatial working memory, you can see various trends across the age span of school, for example,

1. executive and visual–spatial memory processes are used more during learning new mathematical skills/concepts and the phonological loop processes after a skill has been learned.
2. longitudinal studies suggest that some executive processes may be more generic in terms of supporting learning, while others, such as visual–spatial working memory may be more specific to early mathematical learning and verbal processes become more prominent at older ages
3. different aspects of working memory mediate different aspects of mathematical performance for dyscalculic children.
4. working memory is linked with other factors in mathematics learning such as students' ability to use and focus their 'learning attention'. Dyslexic students frequently have difficulty investing attention in what they are learning (Fletcher, 2005; Zentall, 2007). They also have difficulty automatizing what they are learning so that, on later occasions, the knowledge makes a lower demand on thinking space.

An understanding of which aspects of working memory are deficient in children with math difficulties is obscured by a lack of precision in knowing the particular strategies and processes that the child brings to bear on working memory tasks (possibly as a function of age and language) and a theory that links these working memory processes to particular aspects of mathematical learning and performance.

Teaching procedures for mathematics that take account of working memory difficulties

This section examines two aspects of explicit teaching that assists dyscalculic students to improve their mathematics learning ability:

1. An approach to teaching any mathematics idea that includes teacher scaffolding of working memory processes while teaching the idea.
2. The types of working memory strategies you can teach students to use whenever they engage in mathematics learning.

A mathematics teaching regime that includes explicit teacher scaffolding of working memory processes. It is useful to look at the working memory demands in teaching a particular multistep procedure in mathematics. This section examines teaching students to add two fractions with different denominators such as 1/2 +1/3.

Teachers teach this in a variety of ways. Many teachers teach this by having their students manipulate types of quanti typified by those in this diagram and hear the teac

describe how typical tasks are computed while each step is recorded in writing. The students may see the procedure described by the following written statements.

$$1/2 + 1/3 = 3/6 + 2/6$$
$$= 5/6$$

They apply the procedure to a set of examples.

In this section we examine the working memory demands for learning from this type of teaching and explore an alternative approach that assists students to manage the working memory demands.

visual–spatial sketchpad' : retain pictures of ½, 1/3 and how they are moved to show the total

phonological loop : retain the verbal explanations provided by teacher, the description of the steps

'central executive:
- recall names of ½, 1/3 and +,
- calculate new LCD, new numerators and add,
- extract the procedure to apply it to other examples
- know when to use it, that is, how 1 /2+ 1/3= differs from 2/6+3/6 =

the person's existing knowledge
- recall how to 2/6+3/6 =
- recall how to calculate 1/3= 1/6
- store new procedure and when to use it.

We know that some students have difficulty benefiting from this approach to teaching. There is little direct scaffolding of students' working memory use.

An alternative approach cues students to direct their attention to various aspects of the mathematics information in a systematic way and to use various strategies to manage the working memory demands (Munro, 2003, 2000). The teacher uses dialogue in the ways shown in the following diagram to achieve this:

1. encode the task in working memory;
2. stimulate what students already know about this type of task; this provides the existing knowledge base for encoding and representing the new ideas;
3. guide students to link the new task with ones they already can do; this assists them to focus on the particular features of the new task;
4. guide students to encode the type of problem in working memory for later storage in long term memory;
5. stimulate what students already know about key components of the new task; this assists them to retrieve additional relevant aspects of their existing knowledge;
6. stimulate students to use the new links to complete a specific task; this assists them to encode a particular example in context in working memory;
7. repeat with similar particular tasks; this guides students to encode the type of task in working memory;
8. guide the students to identify and describe the new procedure and to practise applying it; this guides students to encode the new procedure in working memory;
9. guide the students to identify when to use the new procedure this guides students to encode in working memory the types of contexts in which they will use the new procedure;
10. guide the students to automatize what they know about how to add two fractions.

Teaching students to add fractions with different denominators (Munro, 2003).

Focus of the teaching	How teacher scaffolds student to use of the components of working memory:
Encode the task in working memory	scaffold students to interpret the task : *Read the task and say what it says. Make a picture of what it says.*
Stimulate what students already know about this type of task.	scaffold students to say what they know about the type of task : *What types of fractions can you add?* *Can you add 2/6+ 3/6 =? Write down some other tasks you can add.*
Guide students to link the new task with ones they can do.	scaffold students to say*: How do these tasks differ from the one we are working on?*
Guide students to encode the type of problem	scaffold students to ask: *Can I make task I can't do like the ones I can do ?*
Stimulate what students already know about key components of the new task.	scaffold students to recall how they can say each fraction in other ways: *What are other fractions that say the same as 1/3 ? Write down some other tasks you can add. Repeat for ½.* *What do these two sets of pictures show ? How could you use what they tell us ?*
Stimulate students to use the new links to complete the particular task.	Scaffold students to see how they can use the alternative names for 1/2 and 1/3 to solve the task 1/2 + 1/3 =. Ask : *Remember we want to find two fractions that are the same as 1/2 and 1/3. Can you find another fraction for 1/2 and 1/3 that have the same denominator ?* Cue the students to rehearse the new links
Repeat with similar tasks.	Scaffold students to use the procedure with similar tasks using the worked example as a model, for example, 2/3 + 1/4 =. Cue them to say what they will do before they begin using the earlier task. Scaffold each step.
Guide the students to identify and describe the new procedure and to	Cue the students to review the tasks they have completed and identify the procedure. Guide them to recognize key aspects of the procedure.

practise applying it.	Have them work through a set of practice tasks. Before they begin each task, ask them to say what they will do.
Guide the students to identify when to use the new procedure	Cue the students to note how these tasks differ from the ones they could already do. Guide them to give a name to the new type of task so that it is distinguished from the earlier type. Ask them to • make up tasks that are not ready and ones that are ready. • classify tasks. • say what they know about the two types of tasks
Guide the students to automatize what they know about how to add two fractions	Cue students to decide rapidly whether an addition of fractions is ready to add and if so to work out the new denominator, numerators and add them.

A teaching regime that includes explicit scaffolding of working memory processes across topics. As well as scaffolding students to meet the working memory demands while teaching a particular multistep procedure in mathematics, teachers can teach students to use working memory strategies across all topics in mathematics.

Key teaching procedures to assist students who have mathematics learning difficulties to encode and manipulate their knowledge in working memory include the following. You can teach students to

1. stimulate explicitly what they already know about the task they will learn. Your teaching can include the relevant mathematics procedures, concepts, the mathematics symbolism and the factual knowledge they will need to use. Students can learn to link the mathematics procedures you will teach with the procedures they already know.

2. say and paraphrase relevant mathematics information such as tasks, number sentences and mathematics ideas. This helps them to 'read' the number sentences into working memory. Have them 'tell themselves' what concrete patterns, pictures of mathematics ideas and mathematics actions show.

3. visualize mathematics ideas, for example, number sentences. Scaffold them to use these strategies and give them time to do so.

4. learn new ideas first through 2 or 3 specific examples and then to extract the procedure. Ask them to talk about what the three examples show.

5. learn each mental action, for example, adding or subtracting as physical actions first and gradually internalize them.

6. use a sequence of self-instructional strategies to guide their way through any task. When they are doing mathematics tasks, teach them to
    - say what a task says;
    - visualize it;
    - say what the solution will be like;
    - categorise the task; say what they will do first, second, …;
    - plan what they will do the task.

7. organise new mathematics ideas into categories. When they have learnt a new mathematics idea, have them recognize instances of it, teach it as a category and teach them a name for the category.

8. review regularly how they learn the ideas and the thinking strategies they used. This helps them learn a repertoire of working memory rehearsal and transformational strategies that they can use on later occasions.

9. say what they have learnt when they have learnt a new mathematics idea. Ask them to say how the new idea is similar to and different from what they already knew. Teach them explicitly to link it with what they knew. This helps them to store the new mathematics knowledge in memory.

10. automatise the new knowledge by teaching independent use of the idea initially. Scaffold them to do more of the idea and gradually remove the scaffolding as they do more of the idea by themselves. Have them practise doing it.

An instructional sequence for teaching these working memory strategies is described in Munro (2011).

Conclusion

In its conclusion the review of working memory and mathematics by Raghubar, et al., (2010) draws attention to our lack of knowledge of the relationship. The authors note the need for a theory of mathematical processing that integrates strategy discovery and selection, the use of mathematical knowledge and specific aspects of working memory. Contemporary developments in neuropsychology could well contribute to such a theory.

Teachers and schools could make an invaluable contribution to such a model. Teachers every day observe how students in their classes respond to the mathematics teaching information. They develop at least an intuitive awareness of the influence of working memory processes on students' on-going mathematical understandings in a way that is denied most researchers and investigators from other disciplines. Insights from how students respond to regular classroom teaching is a potential mine of useful information.

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